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The ρ Meson Light-Cone Distribution Amplitudes of Leading Twist Revisited

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Abstract:

We give a complete re-analysis of the leading twist quark-antiquark light-cone distribution amplitudes of longitudinal and transverse ρ mesons. We derive Wandzura-Wilczek type relations between different distributions and update the coefficients in their conformal expansion using QCD sum rules including next-to-leading order radiative corrections. We find that the distribution amplitudes of quarks inside longitudinally and transversely polarized ρ mesons have a similar shape, which is in contradiction to previous analyses.

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1 Introduction

The theoretical interest in leading twist light-cone distribution amplitudes of hadrons is due to their rôle in the QCD description of hard exclusive processes [1]. In terms of the Bethe-Salpeter wave functions these distributions are defined by keeping track of the momentum fraction x and integrating out the dependence on the transversal momentum $k_\perp$:

$$\phi(x) \sim \int_{k_\perp^2 < \mu^2} d^2 k_\perp \phi(x, k_\perp). \tag{1.1}$$

They describe probability amplitudes to find the hadron in a state with minimum number of Fock constituents and at small transverse separation (which provides an ultraviolet (UV) cut-off). The dependence on the UV cut-off (scale) μ is given by Brodsky-Lepage evolution equations and can be calculated in perturbative QCD, while the distribution amplitudes at a certain low scale provide the necessary non-perturbative input for a rigorous QCD treatment of exclusive reactions with large momentum transfer [2].

Their investigation has been the subject of numerous studies. Chernyak and Zhitnitsky (CZ) have developed an approach to study moments of light-cone distributions using QCD sum rules [3]. Their main conclusion was [4, 5] that the pion and nucleon distribution amplitudes deviate strongly from the asymptotic distributions at large scales, which is a result still under debate. Another result [4, 6] was that the distribution amplitudes of longitudinally and transversely polarized ρ mesons deviate from their asymptotic distributions in opposite directions: the longitudinal distribution is more wide while the transverse one is more narrow. In the further discussion, the pion and nucleon distributions received most attention.

The present paper is devoted to the re-evaluation of ρ meson distributions along the lines of the approach of CZ and is mainly fuelled by newly emerged applications of light-cone distributions for the description of diffractive leptoproduction of vector mesons at HERA [7] and light-cone QCD sum rules for exclusive semileptonic $B \to \rho e\nu$ and radiative $B \to \rho\gamma$ weak decays [8]. The necessity of such an update is due to the following:

First, the old calculations in [4, 6] have used a very low normalization scale $\mu^2 \sim 0.5\,\mathrm{GeV}^2$ and a small value of the QCD coupling. Radiative corrections to the sum rules have been neglected. With the larger values of α_s accepted nowadays the inclusion of the $O(\alpha_s)$ corections to the sum rules is mandatory. The corresponding calculation is a new theoretical result of this paper.

Second, there is a controversy in the sign of the contribution of four-fermion operators to the sum rule for the transverse vector meson as given by CZ [6, 3], and later calculations [9]. This sign difference apparently remained unnoticed and has dramatic consequences for the shape of the distribution.

Third, earlier studies have not spent due attention to distributions of transversely polarized quarks in longitudinally polarized mesons. As first noted in [8], to leading twist accuracy these distributions are given in terms of longitudinal quark spin distributions. We present a detailed derivation of the corresponding relations, the status of which is identical to that of the Wandzura-Wilczek relations [10] between the polarized nuleon structure functions $g_1(x, Q^2)$ and $g_2(x, Q^2)$.

Our presentation is organized as follows. In Sec. 2 we collect relevant definitions and give basic formulas for the expansion of the distribution amplitudes in contributions of

conformal operators, which diagonalize the mixing matrix (Brodsky-Lepage kernels) to leading logarithmic accuracy. Section 3 is devoted to the analysis of QCD sum rules for the distributions in the transversely polarised ρ meson, while Sec. 4 contains the sum rules for the longitudinally polarized ρ meson. The final Sec. 5 contains a summary and some concluding remarks. We also include two appendices with the discussion of more technical issues.

2 The ρ Meson Distribution Amplitudes

2.1 Definitions

We define the light-cone distributions as matrix elements of quark-antiquark non-local gauge invariant operators at light-like separations [3]. For definiteness we consider the ρ^+ meson distributions; the difference to ρ^0 and ω is just a trivial isospin factor in the overall normalization. The complete set of distributions to leading twist accuracy involves four wave functions [8]:

$$\langle 0|\bar{u}(0)\sigma_{\mu\nu}d(x)|\rho^+(p,\lambda)\rangle = i(e^{(\lambda)}_\mu p_\nu - e^{(\lambda)}_\nu p_\mu) f_\rho^\perp \int_0^1 du\, e^{-iupx}\phi_\perp(u,\mu), \tag{2.1}$$

$$\begin{aligned}\langle 0|\bar{u}(0)\gamma_\mu d(x)|\rho^+(p,\lambda)\rangle &= p_\mu \frac{(e^{(\lambda)}x)}{(px)} f_\rho m_\rho \int_0^1 du\, e^{-iupx}\phi_\parallel(u,\mu)\\ &+ \left(e^{(\lambda)}_\mu - p_\mu \frac{(e^{(\lambda)}x)}{(px)}\right) f_\rho m_\rho \int_0^1 du\, e^{-iupx} g_\perp^{(v)}(u,\mu)\,,\end{aligned} \tag{2.2}$$

$$\langle 0|\bar{u}(0)\gamma_\mu\gamma_5 d(x)|\rho^+(p,\lambda)\rangle = -\frac{1}{4}\epsilon_{\mu\nu\rho\sigma}e^{(\lambda)\nu}p^\rho x^\sigma f_\rho m_\rho \int_0^1 du\, e^{-iupx} g_\perp^{(a)}(u,\mu)\,, \tag{2.3}$$

where the gauge factors

$$\mathrm{Pexp}\left[ig\int_0^1 d\alpha\, x^\mu A_\mu(\alpha x)\right]$$

are understood in between the quark fields.

In the above definitions p_μ and $e^{(\lambda)}_\nu$ are the momentum and the polarization vector of the ρ meson, respectively. The integration variable u corresponds to the momentum fraction carried by the quark. The normalization constants f_ρ and $f_\rho^\perp$ (to be detailed later) are chosen in such a way that

$$\int_0^1 du\, f(u) = 1$$

for all the four distributions $f = \phi_\perp, \phi_\parallel, g_\perp^{(v)}, g_\perp^{(a)}$. The functions $\phi_\perp(u,\mu)$ and $\phi_\parallel(u,\mu)$ give the leading twist distributions in the fraction of total momentum carried by the quark in transversely and longitudinally polarized mesons, respectively. The functions $g_\perp^{(v)}(u,\mu)$ and $g_\perp^{(a)}(u,\mu)$ describe transverse polarizations of quarks in the longitudinally polarized mesons and are to a large extent analogous to the spin structure function $g_2(x,Q^2)$ in polarized lepton-nucleon scattering. Similarly to the latter, they receive contributions of

both leading twist 2 and non-leading twist 3, and the twist 2 contributions are related to the longitudinal distribution $\phi_\parallel(u,\mu)$ by Wandzura-Wilczek [10] type relations:

$$g_\perp^{(v),\mathrm{twist}\,2}(u,\mu) = \frac{1}{2}\left[\int_0^u dv \frac{\phi_\parallel(v,\mu)}{\bar v} + \int_u^1 dv \frac{\phi_\parallel(v,\mu)}{v}\right],$$

$$g_\perp^{(a),\mathrm{twist}\,2}(u,\mu) = 2\left[\bar u \int_0^u dv \frac{\phi_\parallel(v,\mu)}{\bar v} + u\int_u^1 dv \frac{\phi_\parallel(v,\mu)}{v}\right]. \tag{2.4}$$

Here and below $\bar v \equiv 1-v$ etc. Eq. (2.4) is derived in App. A and presents one of our main results.

The remaining twist 3 contributions to $g_\perp^{(v)}$, $g_\perp^{(a)}$ can be written in terms of three-particle quark-antiquark-gluon wave functions of transversely polarized vector mesons, cf. [3, 11], and will not be considered in this paper. From now on we will drop the superscript "twist 2", which is always implied.

For some applications it is more convenient to rewrite (2.2) as

$$\begin{aligned}\langle 0|\bar u(0)\gamma_\mu d(x)|\rho^+(p,\lambda)\rangle = {} & p_\mu (e^{(\lambda)}x) f_\rho m_\rho \int_0^1 du\, e^{-iupx}\Phi_\parallel(u,\mu)\\ & + e_\mu^{(\lambda)} f_\rho m_\rho \int_0^1 du\, e^{-iupx} g_\perp^{(v)}(u,\mu),\end{aligned} \tag{2.5}$$

introducing a new distribution function

$$\Phi_\parallel(u,\mu) = \frac{1}{2}\left[\bar u \int_0^u dv \frac{\phi_\parallel(v,\mu)}{\bar v} - u\int_u^1 dv \frac{\phi_\parallel(v,\mu)}{v}\right]. \tag{2.6}$$

Eq. (2.6) follows directly from (2.4) and (2.5) by integration by parts.

2.2 Conformal Expansion and Renormalization

The separation between the quark and the antiquark in Eqs. (2.1)–(2.3) is assumed to be light-like, i.e. $x^2 = 0$. Extracting the leading behaviour of the matrix elements on the light-cone one encounters UV divergences, whose regularization yields a non-trivial scale-dependence which can be described by renormalization group methods [2, 1]. The conformal invariance of QCD at tree level implies that operators with different conformal spin do not mix with each other to leading logarithmic accuracy. For the leading twist distributions $\phi_\perp(u,\mu)$ and $\phi_\parallel(u,\mu)$ it follows that the coefficients a_n of their expansion in Gegenbauer polynomials $C_n^{3/2}(x)$ [12] (that is in contributions of operators with definite conformal spin) are renormalized multiplicatively to that accuracy:

$$\phi(u,\mu) = 6u(1-u)\left[1 + \sum_{n=2,4,\dots} a_n(\mu) C_n^{3/2}(2u-1)\right],$$

$$a_n(\mu) = a_n(\mu_0)\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{(\gamma_{(n)}-\gamma_{(0)})/(2\beta_0)} \tag{2.7}$$

with $\beta_0 = 11 - (2/3)n_f$. The one-loop anomalous dimensions are [13]

$$\gamma_{(n)}^{\parallel} = \frac{8}{3}\left(1 - \frac{2}{(n+1)(n+2)} + 4\sum_{j=2}^{n+1} 1/j\right),$$

$$\gamma_{(n)}^{\perp} = \frac{8}{3}\left(1 + 4\sum_{j=2}^{n+1} 1/j\right). \tag{2.8}$$

The conformal expansion of the distributions $g_\perp^{(v)}$, $g_\perp^{(a)}$ is more complicated and was derived in [8] using the approach of Refs. [14, 15]. We do not repeat the result in this paper, since to leading twist accuracy these distributions are not independent functions, but can be expressed in terms of $\phi_\parallel(u,\mu)$. One finds:

$$g_\perp^{(a)}(u,\mu) = 6u(1-u)\left[1 + \frac{1}{6}a_2^{\parallel}(\mu)\, C_2^{3/2}(\xi) + \ldots\right],$$

$$g_\perp^{(v)}(u,\mu) = \frac{3}{4}(1+\xi^2) + \frac{3}{16}a_2^{\parallel}(\mu)\,(15\xi^4 - 6\xi^2 - 1) + \ldots,$$

$$\Phi_\parallel(u,\mu) = \frac{3}{2}u(1-u)\xi\left[1 + \frac{1}{4}a_2^{\parallel}(\mu)\,(15\xi^2 - 11) + \ldots\right]. \tag{2.9}$$

Here and below we use the notation $\xi = 2u - 1$ as shorthand. The leading contributions in (2.9) agree with the "asymptotic distributions" that were derived in Ref. [11] by a different method, but erroneously identified as being of twist 3.[1]

2.3 Non-Perturbative Input

The decay constants f_ρ, $f_\rho^\perp$ and the coefficients a_n in the Gegenbauer expansion (2.7) are intrinsic hadronic quantities and must be determined either experimentally or by non-perturbative methods. In particular, the decay constant f_ρ is measured [16, 17]:

$$f_{\rho^\pm} = (195 \pm 7)\,\text{MeV}, \qquad f_{\rho^0} = (216 \pm 5)\,\text{MeV}. \tag{2.10}$$

For other quantities, most of the existing information comes from QCD sum rules. In what follows we summarize and update these calculations, taking into account radiative corrections and resolving some discrepancies in earlier studies.

3 Transversely Polarized ρ Mesons

3.1 The Tensor Coupling

The normalization of the leading twist quark-antiquark distribution in the transversely polarized ρ meson is determined by the tensor coupling $f_\rho^\perp$, defined by

$$\langle 0|\bar{u}\sigma_{\mu\nu}d|\rho^+(p,\lambda)\rangle = i(e_\mu^{(\lambda)}p_\nu - e_\nu^{(\lambda)}p_\mu)f_\rho^\perp, \tag{3.1}$$

[1] It is worthwhile to note that these leading terms correspond to the sum of contributions of leading and next-to-leading conformal spin, see [8].

which can be estimated by studying the correlation function of two tensor currents within the framework of QCD sum rules [18]. We refer the reader to the reviews [19] and [3] for detailed explanations of the method; the latter reference deals specifically with the determination of distribution functions. A somewhat troublesome point in studying $f_\rho^\perp$ is that the tensor current also couples to the positive parity $J^{PC} = 1^{+-}$ state $b_1(1235)$ [2] [17]:

$$\langle 0|\bar{u}\sigma_{\mu\nu} d|b_1^+(p,\lambda)\rangle = i\,\epsilon_{\mu\nu}{}^{\alpha\beta} e_\alpha^{(\lambda)} p_\beta f_{b_1}^\perp\,. \tag{3.2}$$

The correlation function of two tensor currents thus contains two Lorentz structures:

$$\begin{aligned}\Pi_{\mu\nu} &= i\int d^4y\, e^{iqy}\,\langle 0|T[\bar{u}(y)\sigma_{\mu\xi}x^\xi d(y)\bar{d}(0)\sigma_{\nu\xi}x^\xi u(0)]|0\rangle \\ &= \frac{1}{q^2}[(qx)(q_\mu x_\nu + q_\nu x_\mu) - (qx)^2 g_{\mu\nu}]\Pi^-(q^2) \\ &\quad + \frac{1}{q^2}[(qx)(q_\mu x_\nu + q_\nu x_\mu) - (qx)^2 g_{\mu\nu} - q^2 x_\mu x_\nu]\Pi^+(q^2)\,. \end{aligned} \tag{3.3}$$

To compactify the Lorentz structure we have contracted the correlation function in two indices by the light-like vector x_μ [3]. The $\Pi^\pm(q^2)$ were calculated in [9] and correspond to intermediate states with positive (negative) parity, respectively:

$$\Pi^\mp(q^2) = -\frac{1}{8\pi^2}q^2\ln\frac{-q^2}{\mu^2}\left[1+\frac{\alpha_s}{3\pi}\Big(\ln\frac{-q^2}{\mu^2}+\frac{7}{3}\Big)\right] - \frac{1}{24q^2}\langle\frac{\alpha_s}{\pi}G^2\rangle + \frac{16\pi}{9q^4}\langle\sqrt{\alpha_s}\bar{q}q\rangle^2\left[\frac{4}{9}\pm 1\right], \tag{3.4}$$

where we used vacuum saturation for the contributions of four-fermion operators.

The correlation function $\Pi^-(q^2)$ can be used to extract the value of $f_\rho^\perp$, see e.g. [9]. Note, however, that it has a higher dimension than the correlation function of vector currents [18], since in the latter case current conservation allows one to include one power of q^2 in the Lorentz structure. The higher dimension significantly reduces the accuracy of the sum rule, as it increases its sensitivity to higher resonances and the continuum. In addition, the sign of the four-quark contribution is reversed, which does not allow to get a stable sum rule for the ρ meson mass in this case, see [9]. To overcome this difficulty, Chernyak and Zhitnitsky suggested to sum contributions of opposite parities. Since one has to assume

$$\Pi^+(0) + \Pi^-(0) = 0$$

to avoid an unphysical singularity at $q^2 = 0$ in Eq. (3.3), it is legitimate to write a dispersion relation for the structure

$$\frac{\Pi^-(q^2)+\Pi^+(q^2)}{q^2} = -\frac{1}{4\pi^2}\ln\frac{-q^2}{\mu^2}\left[1+\frac{\alpha_s}{3\pi}\Big(\ln\frac{-q^2}{\mu^2}+\frac{7}{3}\Big)\right] - \frac{1}{12q^4}\langle\frac{\alpha_s}{\pi}G^2\rangle + \frac{128\pi}{81q^6}\langle\sqrt{\alpha_s}\bar{q}q\rangle^2\,. \tag{3.5}$$

Chernyak and Zhitnitsky speculated [3] that the approximation of local duality for the continuum contributions may be satisfied with better accuracy in sum rules with summation over different parity contributions, and noted that an additional advantage of using

[2] $B(1235)$ in old classification.

(3.5) is that contributions of particular four-fermion operators that are suspected to violate vacuum saturation cancel identically in this case. The price to pay is that the sum rule contains an additional contribution of the $b_1(1235)$ meson; since its mass, however, is very close to the continuum threshold in the ρ meson channel, one may expect that this contamination has a minor effect.

One can thus write down several different sum rules for $f_\rho^\perp$, each of which has its own advantages and disadvantages, and their agreement indicates consistency of the approach. Using (3.5) one obtains

$$e^{-m_\rho^2/M^2}(f_\rho^\perp)^2(\mu) + e^{-m_{b_1}^2/M^2}(f_{b_1}^\perp)^2(\mu) =$$

$$= \frac{1}{4\pi^2}\int_0^{s_0} ds\, e^{-s/M^2}\left(1+\frac{\alpha_s}{\pi}\left[\frac{7}{9}+\frac{2}{3}\ln\frac{s}{\mu^2}\right]\right) - \frac{1}{12M^2}\langle\frac{\alpha_s}{\pi}G^2\rangle - \frac{64\pi}{81M^4}\langle\sqrt{\alpha_s}\bar{q}q\rangle^2 . \quad (3.6)$$

On the other hand, starting from the correlation functions $\Pi^\mp(q^2)$, one gets

$$m_\rho^2 e^{-m_\rho^2/M^2}(f_\rho^\perp)^2(\mu) =$$

$$= \frac{1}{8\pi^2}\int_0^{s_0^\rho} s ds\, e^{-s/M^2}\left(1+\frac{\alpha_s}{\pi}\left[\frac{7}{9}+\frac{2}{3}\ln\frac{s}{\mu^2}\right]\right) + \frac{1}{24}\langle\frac{\alpha_s}{\pi}G^2\rangle + \frac{208\pi}{81M^2}\langle\sqrt{\alpha_s}\bar{q}q\rangle^2 , \quad (3.7)$$

$$m_{b_1}^2 e^{-m_{b_1}^2/M^2}(f_{b_1}^\perp)^2(\mu) =$$

$$= \frac{1}{8\pi^2}\int_0^{s_0^{b_1}} s ds\, e^{-s/M^2}\left(1+\frac{\alpha_s}{\pi}\left[\frac{7}{9}+\frac{2}{3}\ln\frac{s}{\mu^2}\right]\right) + \frac{1}{24}\langle\frac{\alpha_s}{\pi}G^2\rangle - \frac{80\pi}{81M^2}\langle\sqrt{\alpha_s}\bar{q}q\rangle^2 , \quad (3.8)$$

where $s_0^\rho \simeq 1.5\,\mathrm{GeV}^2$ [18] and $s_0^{b_1} \simeq 2.3\,\mathrm{GeV}^2$ [9] are the continuum thresholds in the ρ and b_1 channels, respectively. The continuum threshold s_0 for the "mixed parity" sum rule (3.6) is discussed below. M^2 is the Borel-parameter. Note that the sign of the contribution of four-fermion operators in (3.6) is opposite to the result given in [6, 3]. We have recalculated this contribution and confirm the sign as obtained in [9].

In the numerical analysis we use $\alpha_s(\mu = 1\,\mathrm{GeV}) = 0.56$, i.e. $\Lambda^{(3)}_{\overline{\mathrm{MS}}} = 0.4\,\mathrm{GeV}$, corresponding to the world average $\alpha_s(m_Z) = 0.119$ [17]. For the condensates we take the standard values [18]

$$\langle\frac{\alpha_s}{\pi}G^2\rangle = (0.012\pm0.006)\,\mathrm{GeV}^4 , \qquad \langle\sqrt{\alpha_s}\bar{q}q\rangle^2 = 0.56\,(-0.25\,\mathrm{GeV})^6 . \quad (3.9)$$

The sum rules and the couplings are evaluated at $\mu = 1\,\mathrm{GeV}$. We have checked that changing the scale in the range $\mu^2 = (1-2)\,\mathrm{GeV}^2$ does not have any noticeable effect, provided the extracted couplings are related by renormalization group scaling:[3]

$$f^\perp(1\,\mathrm{GeV}) = \left[\frac{\alpha_s(1\,\mathrm{GeV})}{\alpha_s(\mu)}\right]^{4/27} f^\perp(\mu) .$$

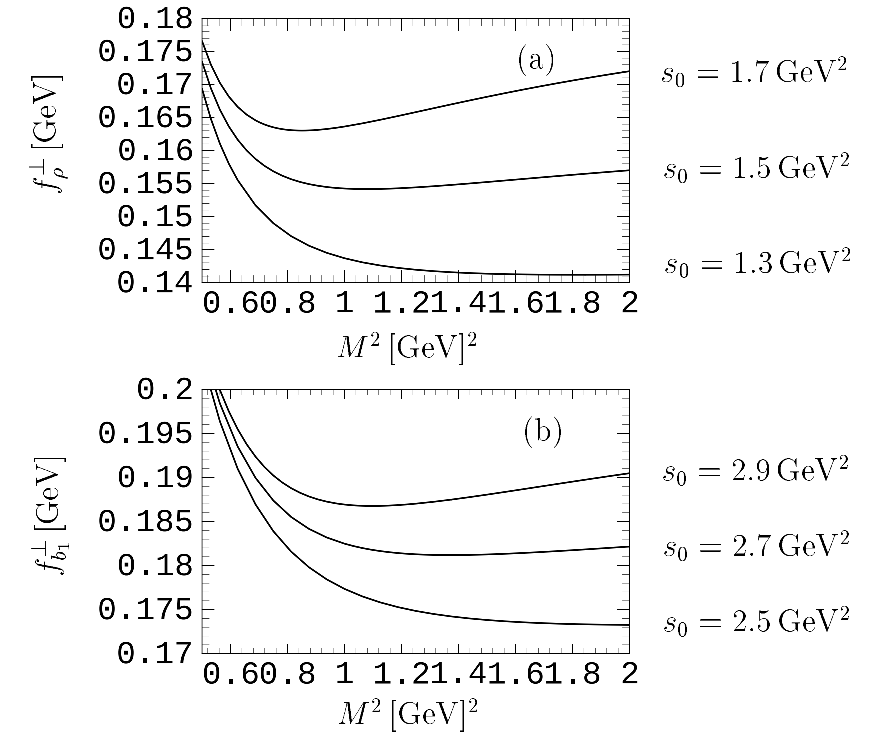

Figure 1: (a) $f_\rho^\perp(1\,\mathrm{GeV})$ from Eq. (3.7) as function of the Borel parameter M^2 for different values of the continuum threshold s_0. (b) The same for $f_{b_1}^\perp(1\,\mathrm{GeV})$ from Eq. (3.8).

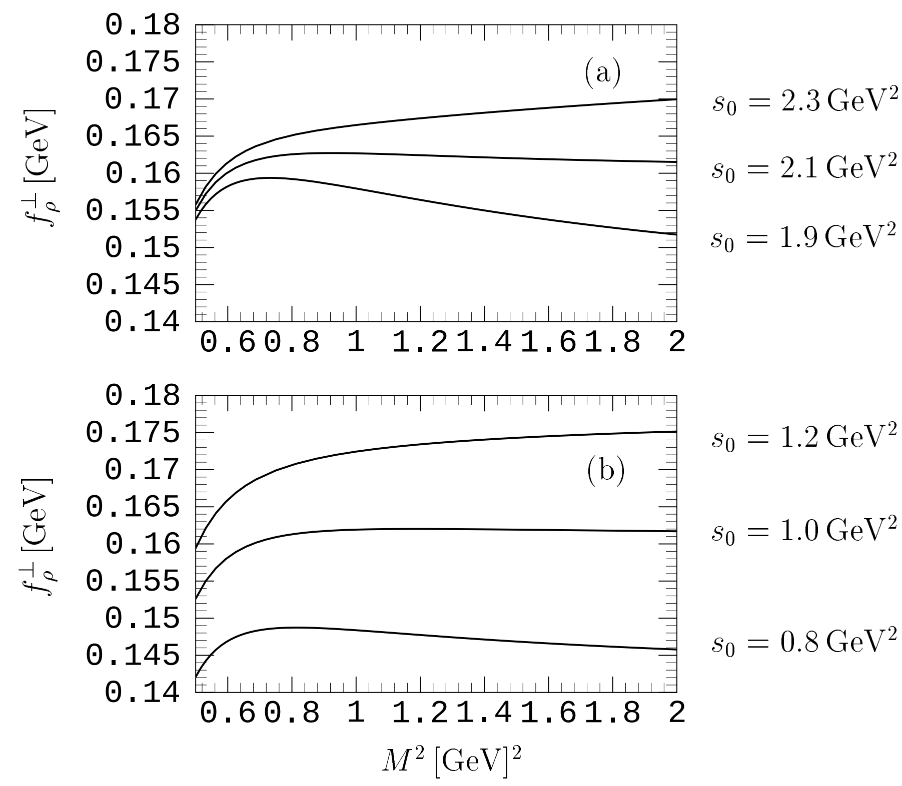

Figure 2: (a) $f_\rho^\perp(1\,\mathrm{GeV})$ from Eq. (3.6) as function of the Borel parameter M^2 for different values of the continuum threshold s_0. $f_{b_1}^\perp(1\,\mathrm{GeV})$ is put to 180 MeV. (b) The same with the b_1 contribution put to the continuum, i.e. $f_{b_1}^\perp = 0$.

We start with the "pure parity" sum rules in Eqs. (3.7) and (3.8). The values of the couplings extracted from these sum rules are shown in Fig. 1a and Fig. 1b as functions of the Borel parameter for several choices of the continuum thresholds. Requiring best stability in the "working window" of the Borel parameter $1 < M^2 < 1.5$ GeV^2, we find

$$f_\rho^\perp = (160 \pm 10)\,\mathrm{MeV}, \qquad s_0^\rho = 1.5\,\mathrm{GeV}^2\,, \tag{3.10}$$

$$f_{b_1}^\perp = 180\,\mathrm{MeV}, \qquad s_0^{b_1} = 2.7\,\mathrm{GeV}^2\,. \tag{3.11}$$

Note that s_0^ρ coincides with the value quoted in [18], while for b_1 we get a somewhat larger value than Ref. [9]. This difference, however, affects the coupling only very slightly: with $s_0^{b_1} = 2.3$ GeV^2 we get $f_{b_1}^\perp = 170\,\mathrm{MeV}$ with a somewhat worse stability. Note also that it is difficult to specify more precisely the value of the continuum threshold s_0^ρ: the stability of the sum rule does not change much with s_0^ρ in the interval (1.3 –1.5) GeV^2 (although the value of $f_\rho^\perp$ does), which is precisely the disadvantage of having a sum rule of high dimension.

Turning to the "mixed parity" sum rule (3.6) we first note that the contribution of b_1 is numerically supressed by the exponential factor $\exp[(m_{b_1}^2 - m_\rho^2)/M^2]$, so that a modest accuracy in $f_{b_1}^\perp$ is sufficient. Using the value in (3.11) as input and requiring best stability of the sum rule (3.6) by varying M^2 and the continuum threshold (see Fig. 2a), we get

$$f_\rho^\perp = (163 \pm 5)\,\mathrm{MeV}, \qquad s_0 = 2.1\,\mathrm{GeV}^2\,. \tag{3.12}$$

The higher value of s_0 (compared to s_0^ρ) is expected, since the part of the continuum contribution coming from b_1 is taken into account explicitly on the left-hand side of the sum rule (3.6).

On the other hand, since the b_1 state is rather wide and its mass is very close to the continuum threshold in the pure 1^{--} channel, it is natural to expect that an equally good fit to the sum rule can be obtained by ignoring this contribution on the left-hand side of (3.6) and fitting the value of the continuum threshold to include it effectively. Remarkably, in this case we find a very similar value for the ρ coupling, see Fig. 2b:

$$f_\rho^\perp = (160 \pm 15)\,\mathrm{MeV}, \qquad s_0^\rho = (1.0 \pm 0.2)\,\mathrm{GeV}^2\,. \tag{3.13}$$

Note that the sum rule now "wants" a much lower value of s_0. It is instructive to observe that the accuracy is now worse since the sum rule remains stable for a rather large interval of s_0. This is natural, since in this case we do not incorporate an additional information about the b_1 meson contribution.

To summarize, we find that the positive parity b_1 meson contributes significantly to the "mixed parity" sum rule, but it is not possible to separate this contribution from the continuum. In effect, the admixture of positive parity states can be described by lowering the duality interval for the ρ meson to 1 GeV. Our final result for the ρ meson tensor coupling is

$$f_\rho^\perp = (160 \pm 10)\,\mathrm{MeV}. \tag{3.14}$$

[3] In this paper we stay consistently within $O(\alpha_s)$ accuracy and do not attempt a renormalization group improvement of sum rules (see e.g. [20]).

This value is by about 20% lower than CZ's result [3] and agrees surprisingly well with an old SU(6) symmetry relation, $f_\rho^\perp = (f_\pi + f_\rho)/2 \approx 0.17\,\mathrm{GeV}$ [21].

As discussed in [6, 3], an alternative method to determine $f_\rho^\perp$ could be to consider the correlation function of the tensor with the vector current, which is not contaminated by positive parity states:

$$\int d^4y\, e^{iqy}\, \langle 0|T[\bar{u}(y)\gamma_\mu d(y)\bar{d}(0)\sigma_{\alpha\beta}u(0)]|0\rangle = [g_{\alpha\mu}q_\beta - g_{\beta\mu}q_\alpha]\,\chi(q^2)\,. \tag{3.15}$$

The correlation function was calculated in [22, 11] and reads:[4]

$$\chi(q^2) = \frac{2\langle\bar{q}q\rangle}{q^2}\Big\{\Big[1 - \frac{2\alpha_s}{3\pi}\Big(2\ln\frac{\mu^2}{-q^2} + 1\Big)\Big] + \frac{m_0^2}{3q^2} + 0\cdot\frac{1}{q^4} + \ldots\Big\} \tag{3.16}$$

Here $m_0^2 \equiv \langle\bar{q}g\sigma Gq\rangle/\langle\bar{q}q\rangle$. Note that the perturbative contribution vanishes to all orders and that the dimension seven operator $\bar{q}G^2q$ has zero coefficient at tree level [22]. The corresponding sum rule reads (cf. [6, 3, 22]):

$$e^{-m_\rho^2/M^2} f_\rho^\perp(\mu)\, f_\rho = -2\langle\bar{q}q\rangle\left[1 + \frac{4}{3}\,\frac{\alpha_s}{\pi}\left(\ln\frac{M^2}{\mu^2} - \gamma_E - \frac{1}{2} - \int_{s_0}^{\infty}\frac{ds}{s}\,e^{-s/M^2}\right)\right.$$
$$\left. - \frac{1}{3}\,\frac{m_0^2}{M^2} + 0\cdot\frac{\langle g_s^2G^2\rangle}{M^4}\right], \tag{3.17}$$

and yields $f_\rho^\perp \approx 200\,\mathrm{MeV}$ as illustrated in Fig. 3. Here we use $f_\rho = 205\,\mathrm{MeV}$, $\langle\bar{q}q\rangle(1\,\mathrm{GeV}) = (-0.25\,\mathrm{GeV})^3$ and $m_0^2 = 0.65\,\mathrm{GeV}^2$ at the scale 1 GeV. The accuracy of this sum rule is, however, not competitive to the ones above: the uncertainty in the quark condensate alone gives a 10% error; in addition, the study in [22] indicates possible large contributions of excited states to this sum rule, e.g. from $\rho'(1600)$. Its significance is, however, that it allows to determine the relative *sign* of $f_\rho^\perp$ and f_ρ, which proves to be positive.

3.2 Deviations from the Asymptotic Form

The deviation of the distribution function from its asymptotic form $\phi_\perp(u) \sim u(1-u)$ is quantified by the coefficients a_n in the expansion (2.7). Since the corresponding anomalous dimensions are ordered with n, one can expect that, at least for large scales μ, only a few first terms are important. The QCD sum rule approach can be used to estimate $a_2^\perp$. The traditional procedure developed by Chernyak and Zhitnitsky is to write down the sum rule for the second moment of the wave function, which is related to $a_2^\perp$ by simple algebra:

$$\int_0^1 du\,(2u-1)^2\phi_\perp(u,\mu) = \frac{1}{5} + \frac{12}{35}\,a_2^\perp(\mu). \tag{3.18}$$

The corresponding sum rule is obtained from the correlation function of the tensor current with the similar operator with two extra covariant derivatives $\bar{u}(y)\sigma_{\mu\xi}x^\xi(i\overleftrightarrow{D}\cdot x)^2d(y)$. We find it more appropriate to consider the sum rules directly for the coefficients in the

[4]The radiative correction to the quark condensate contribution is a new result.

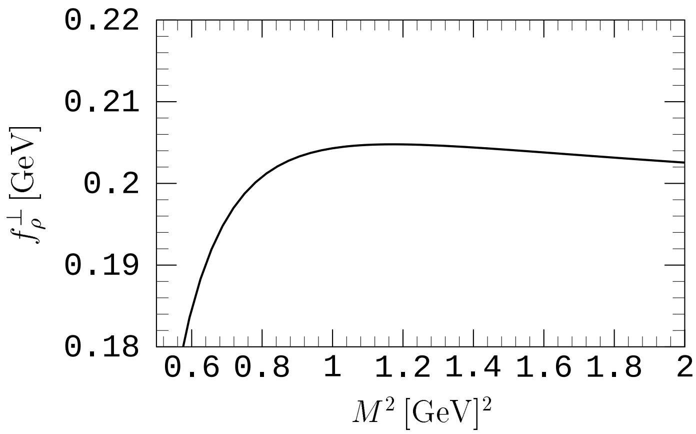

Figure 3: $f_\rho^\perp(1\,\mathrm{GeV})$ from Eq. (3.17) as function of the Borel parameter M^2 for $s_0 = 1.5\,\mathrm{GeV}^2$.

expansion in Gegenbauer polynomials, which in general correspond to correlation functions of the tensor current with the conformal operators

$$\Omega_\perp^{T(n)}(y) = i^n\,(\partial_\cdot)^n \left[\bar{u}(y)\sigma_{\perp\cdot} C_n^{3/2}\left(\frac{\overrightarrow{D}_\cdot - \overleftarrow{D}_\cdot}{\overrightarrow{D}_\cdot + \overleftarrow{D}_\cdot}\right) d(y)\right],$$

where the dots stand for the projection on the light-like vector x_μ and ∂ is the total derivative. Note that one of the indices of the σ matrix is projected onto x_μ, while the other one has to be taken transverse to the (x,q) plane, where q is the ρ meson momentum, see [15] for more details.

As a general property of conformal operators [23] the tree-level perturbative contribution to the corresponding correlation function vanishes (for $n \neq 0$) and the perturbative contribution to the corresponding sum rule starts with order $O(\alpha_s)$. As a result, these sum rules are necessarily less stable than the sum rules for moments, and their accuracy is seemingly worse. The better accuracy of the sum rules for moments is, however, completely illusory since in this case the major contribution comes from the trivial first term in (3.18), corresponding to the asymptotic distribution function, and the contribution of interest is numerically suppressed. Since we should not expect good stability for the sum rule for a_2, we evaluate this sum rule using precisely the same values of the continuum threshold and the same "window" of the Borel parameter as in the sum rules for $f_\rho^\perp$. The instability of the sum rule then gives an estimate of the accuracy of the result.[5]

It is important to note that the necessity to separate the contribution of leading twist does not allow for the separation of contributions of opposite parity in the diagonal sum rules. Indeed, one may try to start from the correlation function like the one in (3.3) with two open Lorentz indices (and with the substitution of one of the tensor currents

[5]It has become a common practice to choose different values of the continuum threshold in the sum rules for different moments. To our point of view, the higher fitted values of s_0 for higher moments $n = 2, 4, \ldots$ generally reflect the increase of the overall mass scale in the correlation function, due to the increasing contribution of higher resonances. This rise has nothing to do with the change of the interval of duality for the ρ meson contribution of interest, which is in fact more likely to *decrease*.

by $\Omega_\mu^{T(n)}(y)$), and try to isolate the negative parity contribution by taking the projection $q^\mu q^\nu \Pi^{T(n)}_{\mu\nu} = (qx)^{n+2}\Pi^{-T(n)}(q^2)$. However, the same projection for the defining Eq. (2.1) vanishes identically since to leading twist accuracy one must put contributions of order $q^2 = m_\rho^2$ to zero. Thus, this projection is in fact saturated by higher twist contributions and is irrelevant for our analysis. Therefore, one cannot get rid of the contribution of states with positive parity and a more convenient correlation function to consider is: [11, 3]

$$i\int d^4y\, e^{iqy}\, \langle 0|T[\bar u(y)\sigma_{\mu\xi}x^\xi d(y)\bar d(0)\sigma^{\mu\xi}x_\xi(i\overleftrightarrow{D}\cdot x)^n u(0)]|0\rangle = -2(qx)^{n+2}\Pi^{T(n)}(q^2). \qquad (3.19)$$

It is easy to check that the trace over Lorentz indices picks up the required transverse components.

The complete results for the sum rules for the coefficients in the Gegenbauer expansion for arbitrary n are given in App. B. Note that in this case the mass scale in the correlation functions rises as $M^2 \sim n^2$ for large n as compared to the increase $M^2 \sim n$ for the moments. This makes the sum rule approach essentially useless for the evaluation of a_n with $n > 2$. For the particular case $n = 2$ we get, using the correlation function (3.19):

$$e^{-m_\rho^2/M^2}(f_\rho^\perp)^2(\mu)\,\frac{18}{7}\,a_2^\perp(\mu)\ + b_1 \text{ meson } =$$

$$= \frac{1}{2\pi^2}\,\frac{\alpha_s(\mu)}{\pi}\,M^2[1-e^{-s_0/M^2}]\cdot\frac{2}{5} + \frac{1}{3M^2}\,\langle\,\frac{\alpha_s}{\pi}\,G^2\,\rangle\ + \frac{64\pi}{9M^4}\,\langle\,\sqrt{\alpha_s}\bar q q\,\rangle^2. \qquad (3.20)$$

This sum rule is equivalent to the sum rule for the second moment considered in [6, 3]

$$e^{-m_\rho^2/M^2}(f_\rho^\perp)^2(\mu)\,\int_0^1 du\,(2u-1)^2\phi_\perp(u,\mu) + b_1 \text{ meson } =$$

$$= \frac{1}{20\pi^2}\int_0^{s_0} ds\, e^{-s/M^2}\left\{1+\frac{\alpha_s}{\pi}\left(\frac{59}{15}+2\ln\frac{s}{\mu^2}\right)\right\} + \frac{1}{36M^2}\,\langle\frac{\alpha_s}{\pi}\,G^2\rangle + \frac{64\pi}{81M^4}\,\langle\sqrt{\alpha_s}\bar q q\rangle^2. \quad (3.21)$$

provided one takes the same value of the continuum threshold as in the sum rule for the tensor coupling (3.6). Note that the sign of the contribution of the four-quark condensate is opposite to the result of [6, 3].[6]

The value of $a_2^\perp$ that follows from the sum rule (3.20) is plotted as a function of the Borel parameter in Fig. 4. Note that we do not have an independent estimate for the contribution of the b_1 meson in this case, so we neglect it and take a low value for the continuum threshold, $s_0 = 1$ GeV^2, on the right-hand side. From this we get as our final result

$$a_2^\perp(\mu = 1\,\mathrm{GeV}) = 0.2 \pm 0.1. \qquad (3.22)$$

This has to be compared with $a_2^\perp(\mu = 1\,\mathrm{GeV}) = -0.17$ from [6, 3]; the difference in sign is mainly due to the opposite sign in the contribution of the four-fermion operators in [6, 3].

We have investigated whether adding the b_1 contribution as a free parameter and requiring best stability in the range $1 < M^2 < 1.5$ GeV^2 could change the result. We have

[6] We have recalculated this contribution and get the opposite sign for all moments, see App. B. For the case $n = 0$ our result agrees with [9].

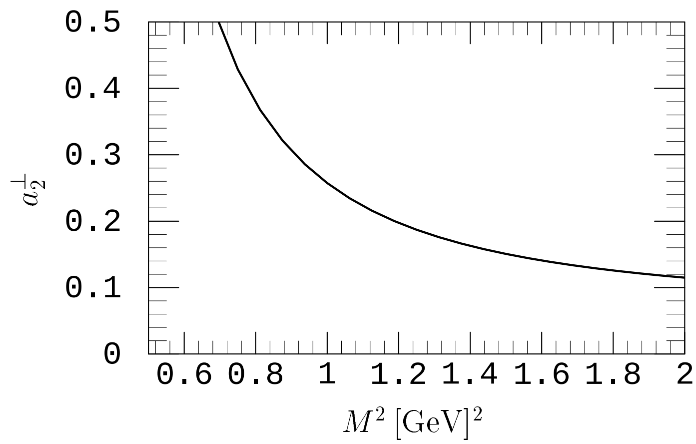

Figure 4: $a_2^\perp(1\,\mathrm{GeV})$ from Eq. (3.20) as function of the Borel parameter M^2 for $s_0 = 1\,\mathrm{GeV}^2$.

also tried to follow the standard procedure to use the sum rule (3.21) with s_0 fitted to get best stability. In both fits the value of $a_2^\perp$ tends to increase by some (30-50)%, but we do not find this evidence significant enough to influence our estimate.

To avoid an admixture of positive parity states, one can consider, instead of (3.19), the correlation function

$$\int d^4y\, e^{iqy}\, \langle 0|T[\bar{u}(y)\gamma_\mu d(y)\bar{d}(0)\sigma_{\alpha\beta}x^\beta(i\overleftrightarrow{D}\cdot x)^n)u(0)]|0\rangle = [g_{\alpha\mu}(qx) - x_\mu q_\alpha]\,(qx)^n\,\chi^{(n)}(q^2). \tag{3.23}$$

The results for $\chi^{(n)}(q^2)$ are available from [24]. The corresponding sum rule for $a_2^\perp$ reads

$$e^{-m_\rho^2/M^2} f_\rho^\perp(\mu)\, f_\rho\, a_2^\perp(\mu) = -\frac{14}{3}\langle\bar{q}q\rangle \left[1 + \frac{29}{18}\frac{\alpha_s}{\pi}\left(\ln\frac{M^2}{\mu^2} - \gamma_E + \text{const.} - \int_{s_0}^{\infty}\frac{ds}{s}\, e^{-s/M^2}\right) - 2\,\frac{m_0^2}{M^2} + \frac{85}{216}\frac{\langle g_s^2G^2\rangle}{M^4}\right], \tag{3.24}$$

where we used vacuum saturation for the contribution of dimension seven. The constant in the radiative correction to the quark condensate contribution is not calculated yet. Unfortunately, due to the large coefficient in front of the contribution of the mixed condensate, its contribution almost identically cancels the leading quark condensate contribution, and the answer depends crucially on the contribution of dimension seven, which is poorly known (it is suspected that vacuum saturation is strongly violated in this case). Thus, from this sum rule one can only get a rough estimate $|a_2^\perp| < 0.5$.

4 Longitudinally Polarized ρ Mesons

Since the decay constant f_ρ is measured experimentally (we use the average value $f_\rho = (205 \pm 10)\,\mathrm{MeV}$ in the numerical analysis), we only need an estimate of the coefficient $a_2^\parallel$ describing the deviation of the distribution $\phi_\parallel$ from its asymptotic form. The corresponding QCD sum rule calculation has been done by Chernyak and Zhitnitsky in Ref. [4]. We update

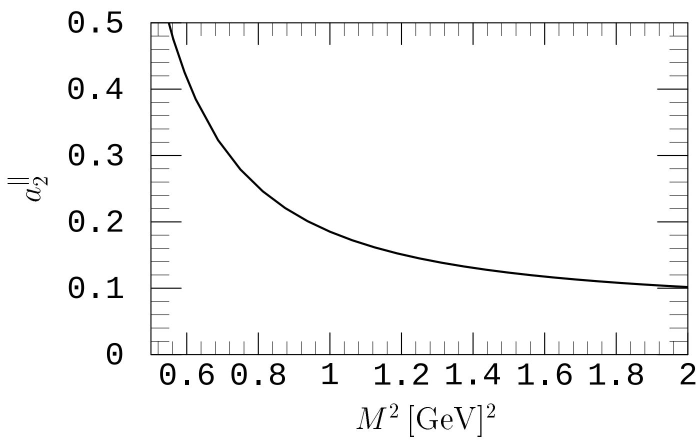

Figure 5: $a_2^\parallel(1\,\mathrm{GeV})$ from Eq. (4.1) as function of the Borel parameter M^2 for $s_0 = 1.5\,\mathrm{GeV}^2$.

this calculation by taking into account radiative $O(\alpha_s)$ corrections and using an up-to-date value of the strong coupling that is slightly larger than the value used in Ref. [4]. The radiative corrections can be extracted from a paper by Gorskii [25], where he calculated the correlation function of two axial vector currents (with extra derivatives), which in perturbation theory and for massless quarks coincides with the vector correlation function. The complete results for arbitrary moments are given in App. B. For $n = 2$ we get the sum rule

$$e^{-m_\rho^2/M^2} f_\rho^2\,\frac{18}{7}\,a_2^\parallel(\mu) = \frac{1}{4\pi^2}\,\frac{\alpha_s(\mu)}{\pi}\,M^2[1-e^{-s_0/M^2}]+\frac{1}{2M^2}\,\langle\,\frac{\alpha_s}{\pi}\,G^2\,\rangle+\frac{32\pi}{9M^4}\,\langle\,\sqrt{\alpha_s}\bar{q}q\,\rangle^2\,, \quad (4.1)$$

which is equivalent to the sum rule for the second moment considered in [4]:

$$e^{-m_\rho^2/M^2} f_\rho^2 \int_0^1 du\,(2u-1)^2\phi_\parallel(u,\mu) =$$
$$= \frac{1}{20\pi^2}\left(1+\frac{5}{3}\frac{\alpha_s}{\pi}\right)M^2\left(1-e^{-s_0/M^2}\right)+\frac{1}{12M^2}\,\langle\frac{\alpha_s}{\pi}\,G^2\rangle+\frac{16\pi}{81M^4}\,\langle\sqrt{\alpha_s}\bar{q}q\rangle^2. \qquad (4.2)$$

With the same input parameters as in Sec. 3, the numerical analysis yields (see Fig. 5):

$$a_2^\parallel(\mu = 1\,\mathrm{GeV}) = 0.18 \pm 0.10. \qquad (4.3)$$

This is in perfect agreement with the original estimate $a_2^\parallel(\mu = 1.1\,\mathrm{GeV}) \simeq 0.18$ [4]. It also coincides within the errors with our result for $a_2^\perp$, Eq. (3.22), which means that the distribution amplitudes $\phi_\parallel$ and $\phi_\perp$ are similar.

5 Summary and Conclusions

Extending earlier studies [4, 6, 3], we have performed a re-analysis of ρ meson quark-antiquark light-cone distribution amplitudes of leading twist. In general, their complete set consists of four independent functions, but we have shown that to our (twist 2) accuracy

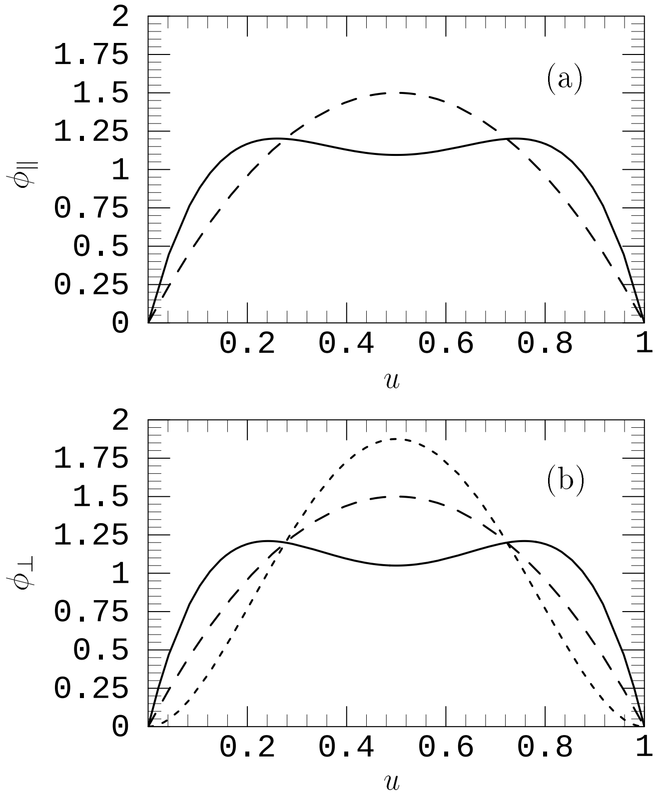

Figure 6: Final results for the wave functions $\phi_\parallel$ (a) and $\phi_\perp$ (b) at $\mu = 1\,\mathrm{GeV}$ (solid lines). Long dashes: asymptotic wave functions, short dashes: $\phi_\perp$ according to CZ [3].

the distributions of transversely polarized quarks in the longitudinally polarized ρ mesons can be related to the distributions of longitudinally polarized quarks. The theoretical status of these relations is identical to the status of the Wandzura-Wilczek relation [10] between the polarized structure functions of the nucleon, $g_1(x, Q^2)$ and $g_2(x, Q^2)$.

We have given a detailed re-analysis of the QCD sum rules for the first two moments of the distribution amplitudes, complementing existing sum rules by the calculation of radiative corrections. Our final results for the distribution amplitudes of quarks in longitudinally polarized and transversely polarized ρ mesons are shown in Fig. 6a and b, respectively. The solid curves correspond to the distributions calculated using the parameters specified in the text, the dashed curves show the asymptotic distributions.

We deviate from the results of [6, 3] mainly in the shape of the distribution amplitude for the transversely polarized ρ meson, see the short-dashed curve in Fig. 6b. We find that the distributions in longitudinally and transversely polarized ρ mesons coincide to our accuracy, whereas in [6, 3] a significant difference has been claimed. This contradiction is largely due to an opposite sign of the contribution of four-fermion operators in the corresponding sum rule. We note that the sign as given in [6, 3] also contradicts an independent calculation in Ref. [9], which apparently remained unnoticed. One more consequence of this sign difference is that our result for the tensor coupling $f_\rho^\perp$ is 20% lower than in [6, 3].

A discussion of phenomenological consequences of our results goes beyond the tasks of this paper. Since in hard exclusive processes one typically deals with integrals over quark

distributions of type

$$f_\rho \int_0^1 du \, \frac{\phi(u,Q)}{u(1-u)} = 6 f_\rho \, [1 + a_2(Q) + \ldots] \, ,$$

the change in shape of the transverse ρ distribution suggested by the results of this paper may increase the rate of the production of transversely polarized ρ mesons by a factor two. The consequences for exclusive semileptonic and radiative B decays will be considered in a separate publication [26].

Acknowledgements: We gratefully acknowledge the kind hospitality of the DESY Theory Group, where this work was finished. We also would like to thank V. Chernyak for correspondance.

Appendix A: Transverse Spin Distributions

The derivation of relations between longitudinal and transverse quark spin distributions in the longitudinally polarized ρ meson is in principle straightforward and can be done similarly to the classical Wandzura-Wilczek analysis for polarized leptoproduction [10]. The major difference is that one must include operators with total derivatives and that higher twist operators corresponding to total derivatives of lower twist operators cannot be neglected.

It is convenient to consider the relevant non-local operator at symmetric quark-antiquark separations:

$$\bar{u}(-x)\gamma_\mu d(x) = \sum_n x^{\mu_1} \ldots x^{\mu_n} \frac{1}{n!} \bar{u}(0) \overleftrightarrow{D}_{\mu_1} \ldots \overleftrightarrow{D}_{\mu_n} \gamma_\mu d(0) \, . \tag{A.1}$$

Since $x^2 = 0$ the arising local operators are traceless (contractions of the type $g_{\mu\mu_k}$ vanish by the equations of motion), but not fully symmetric in Lorentz indices because of the distinguished index μ. Therefore, they contain a mixture of contributions of twist 2 and twist 3, which have to be separated:

$$\bar{u}(-x)\gamma_\mu d(x) = \Big[\bar{u}(-x)\gamma_\mu d(x)\Big]_{\text{twist 2}} + \Big[\bar{u}(-x)\gamma_\mu d(x)\Big]_{\text{twist 3}} . \tag{A.2}$$

The leading twist 2 contribution by definition contains contributions of symmetrized operators:

$$\Big[\bar{u}(-x)\gamma_\mu d(x)\Big]_{\text{twist 2}} \equiv$$

$$\equiv \sum_{n=0}^{\infty} \frac{x^{\mu_1} \ldots x^{\mu_n}}{n!} \bar{u}(0) \left\{ \frac{1}{n+1} \overleftrightarrow{D}_{\mu_1} \ldots \overleftrightarrow{D}_{\mu_n} \gamma_\mu + \frac{n}{n+1} \overleftrightarrow{D}_\mu \overleftrightarrow{D}_{\mu_1} \ldots \overleftrightarrow{D}_{\mu_{n-1}} \gamma_{\mu_n} \right\} d(0) \, . \tag{A.3}$$

Fortunately, the sum can be re-expressed in terms of a non-local operator [27],

$$\Big[\bar{u}(-x)\gamma_\mu d(x)\Big]_{\text{twist 2}} = \int_0^1 dv \, \frac{\partial}{\partial x_\mu} \bar{u}(-vx) \, \not{x} d(vx) \, , \tag{A.4}$$

which is easily verified by expanding. An identical expression is valid for the non-local operator with an additional γ_5 in between the quarks.

Using the equations of motion, the difference $\bar{u}(-x)\gamma_\mu d(x) - [\bar{u}(-x)\gamma_\mu d(x)]_{\rm twist\,2}$ can be written in terms of operators containing total derivatives and quark-antiquark-gluon operators, see [27]. Neglecting quark masses, one finds:

$$\Big[\bar{u}(-x)\gamma_\mu d(x)\Big]_{\rm twist\,3} = -\,g_s\int_0^1 du\int_{-u}^{u} dv\,\bar{u}(-ux)\Big[u\tilde{G}_{\mu\nu}(vx)x^\nu \not{x}\gamma_5 - ivG_{\mu\nu}(vx)x^\nu \not{x}\Big]d(ux)$$

$$+\,i\epsilon_\mu{}^{\nu\alpha\beta}\int_0^1 udu\, x_\nu\partial_\alpha\Big[\bar{u}(-ux)\gamma_\beta\gamma_5 d(ux)\Big]\,,$$

$$\Big[\bar{u}(-x)\gamma_\mu\gamma_5 d(x)\Big]_{\rm twist\,3} = -g_s\int_0^1 du\int_{-u}^{u} dv\,\bar{u}(-ux)\Big[u\tilde{G}_{\mu\nu}(vx)x^\nu \not{x} - ivG_{\mu\nu}(vx)x^\nu \not{x}\gamma_5\Big]d(ux)$$

$$+\,i\epsilon_\mu{}^{\nu\alpha\beta}\int_0^1 udu\, x_\nu\partial_\alpha\Big[\bar{u}(-ux)\gamma_\beta d(ux)\Big]\,, \qquad \text{(A.5)}$$

where $G_{\mu\nu}$ is the gluon field strength, $\tilde{G}_{\mu\nu} = (1/2)\epsilon_{\mu\nu\alpha\beta}G^{\alpha\beta}$, and ∂_α is the derivative over the total translation:

$$\partial_\alpha\Big[\bar{u}(-ux)\gamma_\beta d(ux)\Big] \equiv \frac{\partial}{\partial y_\alpha}\Big[\bar{u}(-ux+y)\gamma_\beta d(ux+y)\Big]\Big|_{y\to 0}\,. \qquad \text{(A.6)}$$

Note that (A.4) and (A.5) are exact operator relations. Taking the matrix element between the vacuum and the ρ meson state, we get

$$\langle 0|\Big[\bar{u}(-x)\gamma_\mu d(x)\Big]_{\rm twist\,2}|\rho^+(p,\lambda)\rangle =$$

$$= \int_0^1 dv\,\frac{\partial}{\partial x_\mu}\langle 0|\bar{u}(-vx)\not{x}d(vx)|\rho^+(p,\lambda)\rangle \; = \int_0^1 dv\,\frac{\partial}{\partial x_\mu}(e^{(\lambda)}x)f_\rho m_\rho\int_0^1 du\, e^{-i\xi vpx}\phi_\parallel(u)$$

$$= e^{(\lambda)}_\mu f_\rho m_\rho\int_0^1 dv\int_0^1 du\, e^{-i\xi vpx}\phi_\parallel(u) - ip_\mu(e^{(\lambda)}x)f_\rho m_\rho\int_0^1 dv\, v\int_0^1 du\,\xi e^{-i\xi vpx}\phi_\parallel(u)$$

$$= p_\mu\frac{(e^{(\lambda)}x)}{(px)}\,f_\rho m_\rho\int_0^1 du\, e^{-i\xi px}\phi_\parallel(u) + \left(e^{(\lambda)}_\mu - p_\mu\frac{(e^{(\lambda)}x)}{(px)}\right)f_\rho m_\rho\int_0^1 du\int_0^1 dv\, e^{-i\xi vpx}\phi_\parallel(u)\,, \qquad \text{(A.7)}$$

where $\xi \equiv 2u-1$ and to arrive at the last line we have used

$$\int_0^1 dv\, v\int_0^1 du\,\xi\, e^{-i\xi vpx}\phi_\parallel(u) = \frac{i}{px}\int_0^1 dv\, v\int_0^1 du\,\frac{\partial}{\partial v}e^{-i\xi vpx}\phi_\parallel(u)$$

$$= \frac{i}{px}\int_0^1 du\,\phi_\parallel(u)\Big[e^{-i\xi px} - \int_0^1 dv\, e^{-i\xi vpx}\Big]. \qquad \text{(A.8)}$$

Note that the matrix element of the twist 2 operator produces both Lorentz structures, and hence $g_\perp^{(v)}(u,\mu)$ is nonzero to this accuracy.

Specific for the kinematics in exclusive processes is the generation of an additional twist 2 contribution by twist 3 operators proportional to the total derivative $\partial_\alpha \to -ip_\alpha$, which

would vanish in deep inelastic scattering. Taking the matrix element for the twist 3 operator in the first of Eqs. (A.5) and neglecting three-particle quark-antiquark-gluon distributions of twist 3 [11] we get

$$\langle 0|\Big[\bar u(-x)\gamma_\mu d(x)\Big]_{\rm twist\,3}|\rho^+(p,\lambda)\rangle =$$

$$= -\frac{1}{2}(px)^2\left(e^{(\lambda)}_\mu - p_\mu\frac{(e^{(\lambda)}x)}{(px)}\right)f_\rho m_\rho\int_0^1 v^2 dv\int_0^1 du\, e^{-i\xi vpx} g_\perp^{(a)}(u,\mu)\,. \tag{A.9}$$

Since, on the other hand,

$$\langle 0|\bar u(-x)\gamma_\mu d(x)|\rho^+(p,\lambda)\rangle = p_\mu\frac{(e^{(\lambda)}x)}{(px)}f_\rho m_\rho\int_0^1 du\, e^{-i\xi px}\phi_\parallel(u,\mu)$$

$$+\left(e^{(\lambda)}_\mu - p_\mu\frac{(e^{(\lambda)}x)}{(px)}\right)f_\rho m_\rho\int_0^1 du\, e^{-i\xi px} g_\perp^{(v)}(u,\mu)\,, \tag{A.10}$$

we obtain relations between $g_\perp^{(v)}(u,\mu)$, $g_\perp^{(a)}(u,\mu)$ and $\phi_\parallel(u,\mu)$ by comparing the Lorentz structures. At this stage it is convenient to introduce the moments

$$M_n^\parallel = \int_0^1 du\,\xi^n\phi_\parallel(u,\mu)\,,\quad M_n^v = \int_0^1 du\,\xi^n g_\perp^{(v)}(u,\mu)\,,\quad M_n^a = \int_0^1 du\,\xi^n g_\perp^{(a)}(u,\mu)\,. \tag{A.11}$$

Expanding (A.7), (A.9), (A.10) in powers of (px), we get

$$M_n^v = \frac{1}{2}\frac{n(n-1)}{n+1}M_{n-2}^a + \frac{1}{n+1}M_n^\parallel \tag{A.12}$$

Similar manipulations with the axial-vector operator (2.3) produce one more relation

$$\frac{1}{2}M_n^a = \frac{1}{n+2}M_n^v. \tag{A.13}$$

Note that the contribution of the leading twist operator $\langle 0|[\bar u(-x)\gamma_\mu\gamma_5 d(x)]_{\rm twist\,2}|\rho^+(p,\lambda)\rangle$ vanishes identically in this case, and the answer is generated entirely by twist 3 operators, which are reduced to total derivatives.

Combining (A.12) and (A.13) we get a simple recurrence relation,

$$(n+1)M_n^v = (n-1)M_{n-2}^v + M_n^\parallel\,, \tag{A.14}$$

the solution of which yields the first relation in (2.4). The second one then follows from (A.13) after some algebra.

Appendix B: QCD Sum Rules for Arbitrary Moments

In this appendix we collect some more definitions and give the sum rules for the Gegenbauer moments a_n of the longitudinal and transversal ρ meson distribution amplitudes for arbitrary n.

We first relate the a_n to hadronic matrix elements of local operators. To leading logarithmic accuracy, the relevant multiplicatively renormalizable operators are:

$$\Omega^{V(n)}(y) = \sum_{j=0}^{n} c_{n,j}(ix\partial)^{n-j}\bar{u}(y)\,\not{x}\,(ix\overleftrightarrow{D})^j d(y)\,,$$

$$\Omega^{T(n)}_{\mu}(y) = \sum_{j=0}^{n} c_{n,j}(ix\partial)^{n-j}\bar{u}(y)\sigma_{\mu\nu}x^{\nu}(ix\overleftrightarrow{D})^j d(y)\,, \tag{B.1}$$

where x_μ is a light-like vector, $\sigma_{\mu\nu} = (i/2)[\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu]$ and $\overleftrightarrow{D}_\mu = \overrightarrow{\partial}_\mu - \overleftarrow{\partial}_\mu - 2igA^a_\mu(y)\lambda^a/2$. The $c_{n,k}$ are the coefficients of the Gegenbauer polynomials such that $C^{3/2}_n(x) = \sum c_{n,k}x^k$. Expanding the defining relations for ϕ, Eq. (2.1)–(2.3), around the light-cone, one finds

$$\begin{aligned}
\langle 0|\Omega^{V(n)}(0)|\rho\rangle &= (px)^n\, f_\rho m_\rho (ex) \int_0^1 du\, C^{3/2}_n(2u-1)\,\phi_\parallel(u,\mu) \\
&= (px)^n\, f_\rho m_\rho (ex)\, \frac{3(n+1)(n+2)}{2(2n+3)}\, a^\parallel_n(\mu)\,, \\
\langle 0|\Omega^{T(n)}_\mu(0)|\rho\rangle &= (px)^n\, i f^\perp_\rho (e_\mu(px) - p_\mu(ex)) \int_0^1 du\, C^{3/2}_n(2u-1)\,\phi_\perp(u,\mu) \\
&= (px)^n\, i f^\perp_\rho (e_\mu(px) - p_\mu(ex))\, \frac{3(n+1)(n+2)}{2(2n+3)}\, a^\perp_n(\mu).
\end{aligned} \tag{B.2}$$

The QCD sum rules [18] are obtained by matching the representation in terms of hadronic states to the operator product expansion in the Euclidian region for the correlation functions

$$\begin{aligned}
(qx)^{n+2}\Pi^{V(n)}(q^2) &= i\int d^4y\, e^{iqy}\, \langle 0|T\Omega^{V(n)}(y)\Omega^{\dagger V(0)}(0)\rangle, \\
-2(qx)^{n+2}\Pi^{T(n)}(q^2) &= i\int d^4y\, e^{iqy}\, \langle 0|T\Omega^{T(n)}_\mu(y)\Omega^{\dagger T(0)\mu}(0)\rangle.
\end{aligned} \tag{B.3}$$

Note that the contraction over μ in the second relation automatically projects onto the transverse component $\Omega^{T(n)}_\perp$, which is a conformal invariant operator, whereas $\Omega^{T(n)}_\mu$ is not. We find the following sum rules for $a^\perp_n$ (for even n):

$$\begin{aligned}
& e^{-m_\rho^2/M^2}(f^\perp_\rho)^2(\mu)\,\frac{3(n+1)(n+2)}{2(2n+3)}\,a^\perp_n(\mu) = \\
&= \frac{1}{2\pi^2}\,\frac{\alpha_s(\mu)}{\pi}\,M^2[1-e^{-s_0/M^2}]\int_0^1 du\,u\bar{u}\,C^{3/2}_n(2u-1)\left(\ln u + \ln\bar{u} + \ln^2\frac{u}{\bar{u}}\right) \\
&\quad + \frac{1}{24M^2}\,\langle\frac{\alpha_s}{\pi}G^2\rangle\,(n^2+3n-2) + \frac{8\pi}{81M^4}\,\langle\sqrt{\alpha_s}\bar{q}q\rangle^2\,(n-1)(n+1)(n+2)(n+4)\,.
\end{aligned} \tag{B.4}$$

The radiative correction in (B.4) is a new result. Similarly, we obtain for $a_n^\parallel$:

$$e^{-m_\rho^2/M^2} f_\rho^2 \frac{3(n+1)(n+2)}{2(2n+3)} a_n^\parallel(\mu) \; = $$

$$= \frac{3}{4\pi^2} \frac{\alpha_s(\mu)}{\pi} M^2 [1-e^{-s_0/M^2}]\, r_n^\parallel + \frac{1}{24M^2} \langle \frac{\alpha_s}{\pi} G^2 \rangle (n+1)(n+2)$$

$$+ \frac{8\pi}{81M^4} \langle \sqrt{\alpha_s}\bar{q}q \rangle^2 (n+1)(n+2)(n^2+3n-7)\,. \tag{B.5}$$

In this case a compact answer for the radiative corrections as in (B.4) is not available, but the $r_n^\parallel$ are related to the radiative corrections to the axial vector correlation function (with extra derivatives) and for arbitrary n can be expressed in terms of the coefficients A_k' calculated in [25]:

$$r_n^\parallel = \sum_{k=0}^n c_{n,k} \frac{A_k'}{(k+1)(k+3)}\,. \tag{B.6}$$

In particular

$$A_0' = 1, \qquad A_2' = \frac{5}{3}, \qquad A_4' = \frac{59}{27}, \qquad A_6' = \frac{353}{135},$$

$$r_0^\parallel = \frac{1}{3}, \qquad r_2^\parallel = \frac{1}{3}, \qquad r_4^\parallel = \frac{1}{6}, \qquad r_6^\parallel = \frac{83}{810}.$$

For completeness and for comparison with [3], we also give the sum rules for the moments $\langle \xi^n \rangle = \int du\, \xi^n \phi(u,\mu)$:

$$(f_\rho^\perp)^2(\mu) \langle \xi^n \rangle_\perp(\mu)\, e^{-m_\rho^2/M^2} = \frac{3}{2\pi^2} \int_0^{s_0} ds \int_0^1 du\, e^{-s/M^2}\, u\bar{u}\, (2u-1)^n \Bigg\{ 1 + \frac{\alpha_s}{3\pi} \Bigg(6 - \frac{\pi^2}{3} + 2\ln\frac{s}{\mu^2}$$

$$+ \ln u + \ln\bar{u} + \ln^2 \frac{u}{\bar{u}} \Bigg) \Bigg\} + \frac{n-1}{n+1} \frac{1}{12M^2} \langle \frac{\alpha_s}{\pi} G^2 \rangle + \frac{64\pi}{81M^4} (n-1) \langle \sqrt{\alpha_s}\bar{q}q \rangle^2, \tag{B.7}$$

$$f_\rho^2 \langle \xi^n \rangle_\parallel\, e^{-m_\rho^2/M^2} = \frac{3}{4\pi^2 (n+1)(n+3)} \left(1 + \frac{\alpha_s}{\pi} A_n' \right) M^2 \left(1 - e^{-s_0/M^2} \right)$$

$$+ \frac{1}{12M^2} \langle \frac{\alpha_s}{\pi} G^2 \rangle + \frac{16\pi}{81M^4} (4n-7) \langle \sqrt{\alpha_s}\bar{q}q \rangle^2. \tag{B.8}$$

Note the difference in sign in the last term in Eq. (B.7) with respect to (4.25) in Ref. [3].

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